Foundation and Higher GCSE Exam-Style Questions

Algebra 4-5

1. Fully simplify:

   a. $r \times r \times r \times r$

   b. $3a + 4a^2 + 2a - a^2$

   c. $(2p^5q)^3$

   d. $\frac{3x^2y}{6xy^2}$

2. a. $y = 8n + 11$

   Work out the value of y when $n = 5$

   b. $y = 3n - 4$

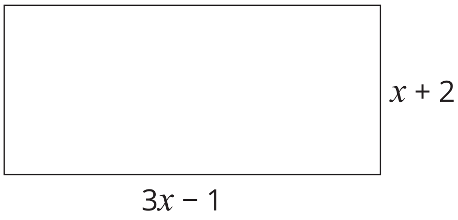
   Work out the value of n when $y = 8$

3. $v = ut + \frac{1}{2}at^2$

   Find the value of v when $u = 0$, $a = 5$ and $t = 3$

BEYOND MATHS

4. The diagram shows a rectangle. All dimensions are in centimetres.



a. Write an expression in terms of x for the perimeter of the rectangle. Give your answer in its simplest form.

b. Given that the perimeter of the rectangle is 54cm, find the value of x.

5. Alexis has n marbles. Ben has three more marbles than Alexis. Carly has twice the amount of marbles that Alexis and Ben have between them. Write an expression in terms of n for the total number of marbles that the three children have. Give your answer in its simplest form.

6. Solve:

a. $3x = 15$

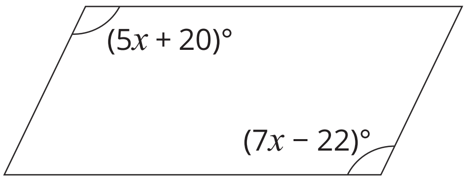
b. $x - 7 = 11$

c. $\frac{x}{4} = 8$

d. $3(2x + 1) = 12$

e. $5x - 1 = 2 - x$

7. The diagram shows a parallelogram. Find the size of the smallest angle in the parallelogram.



8. Rearrange to make x the subject:

a. $p = 3x + r$

b. $y = \frac{1}{2}x + 5$

c. $A = \pi x^2$

9. Expand and simplify:

a. $4x(x + 7)$

b. $3(2p + 1) - 2(p - 7)$

c. $(y + 5)(y - 2)$

d. $(w + 2)^2$

10. Factorise fully:

a. $8x - 12$

b. $9p^2 + 6p$

c. $8ab + 10b + 12ab^2$

d. $x^2 + 9x + 20$

e. $y^2 + 3y - 18$

f. $4x^2 - 9$

BEYOND MATHS

11. Solve the simultaneous equations:

a. $3x + y = 8$
$x + y = 2$

b. $4x + 3y = 8$
$2x - y = -1$

12. Two families are going to the cinema.
Ben buys two adult tickets and two child tickets and pays a total of £21.20.
Charlie buys one adult ticket and three child tickets and pays a total of £17.60.
Work out the cost of buying one adult ticket and one child ticket.

13. A linear sequence has n^{th} term $4n - 3$
Work out the value of the eighth term in the sequence.

14. The first three terms of a geometric sequence are 400, 200, 100.
Write down the next two terms of this sequence.

15. Find the n^{th} terms for the linear sequences whose first four terms are:

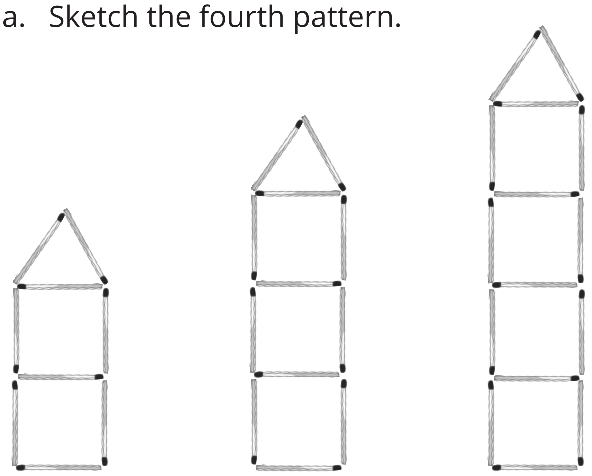
a. 3, 6, 9, 12

b. 5, 11, 17, 23

c. 12, 10, 8, 6

16. Zainab builds a sequence of patterns using matchsticks.
The first three patterns are shown below.

a. Sketch the fourth pattern.



b. Work out how many matchsticks she would need for the 20^{th} pattern.

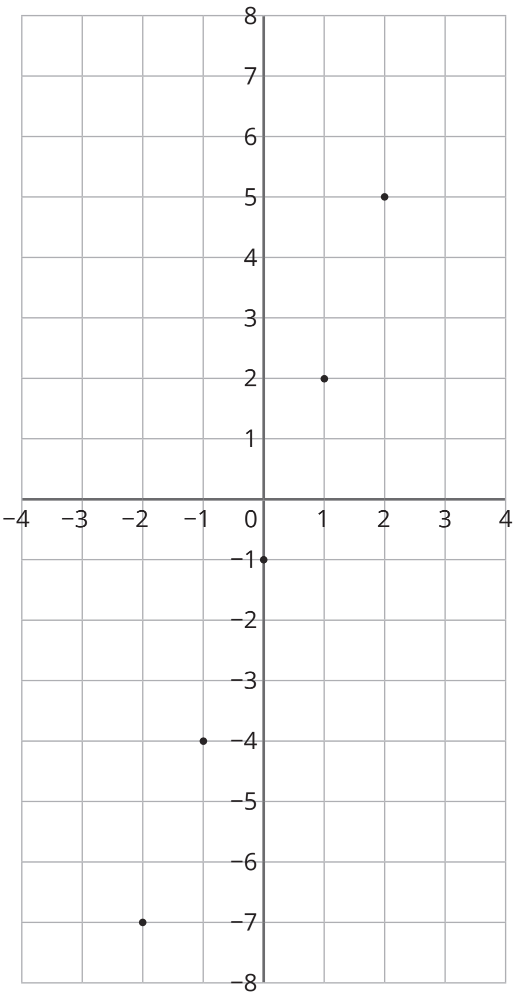
17. Work out the first three terms of the sequence whose n^{th} term is $10 - n^2$

18. The first four terms of a Fibonacci sequence are 1, 1, 2, 3. Work out the value of the 7th term in this sequence.

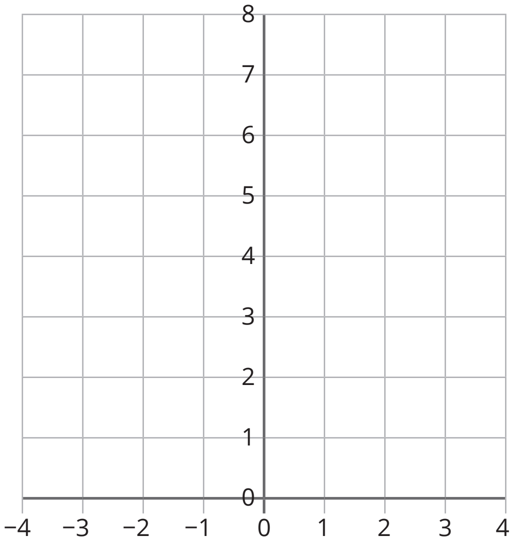
19. a. Complete the table of values for $y = 3x - 1$

x	−2	−1	0	1	2
y	−7			2	

b. On the grid, draw the graph of $y = 3x - 1$ for $-2 \le x \le 2$



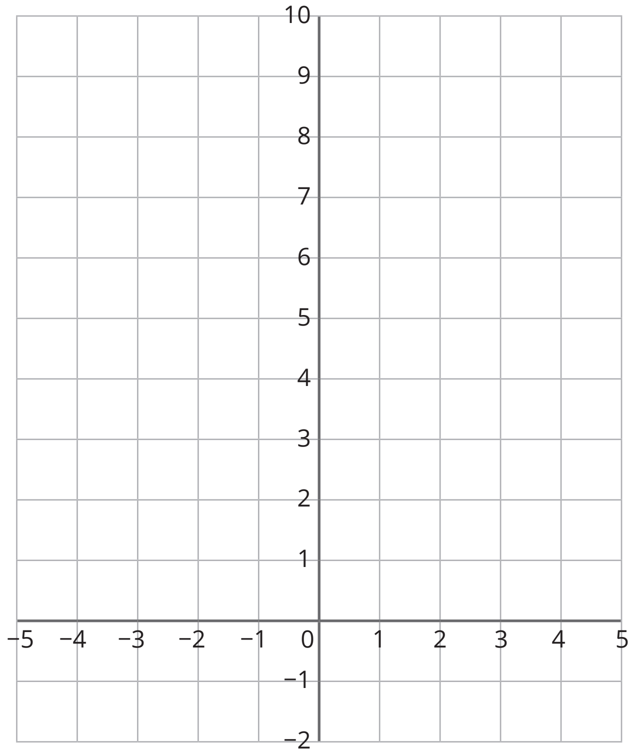
20. On the grid, draw the graph of $x + y = 4$ for $-2 \leq x \leq 2$



21. a. Complete the table of values for $y = x^2 - 2x$

x	−2	−1	0	1	2	3
y		3			0	

b. On the grid, draw the graph of $y = x^2 - 2x$ for $-2 \leq x \leq 3$

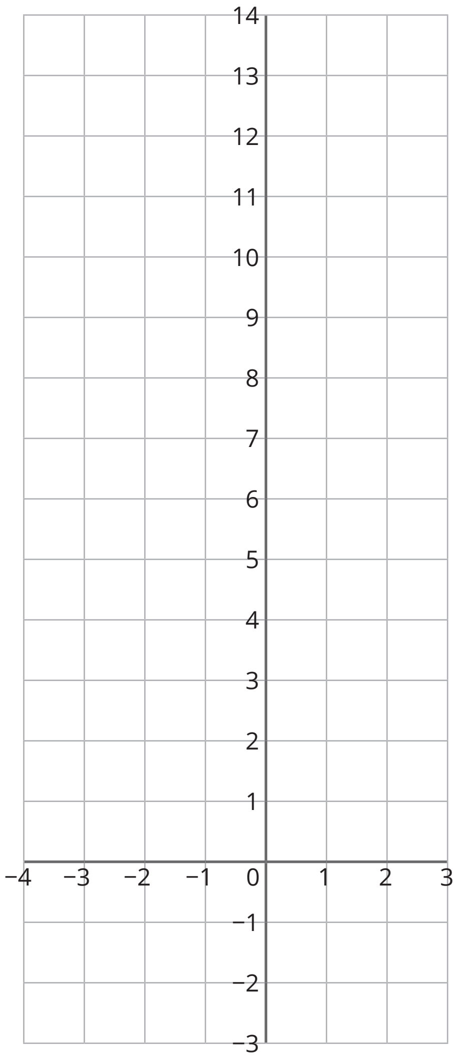


c. Write down the coordinates of the turning point of this graph.

22. a. Complete the table of values for $y = 2x^2 + 3x - 1$

x	−3	−2	−1	0	1	2
y	8				4	

b. On the grid, draw the graph of $y = 2x^2 + 3x - 1$ for $-3 \le x \le 2$



c. Use your graph to estimate all the solutions to the equation $2x^2 + 3x - 1 = 0$

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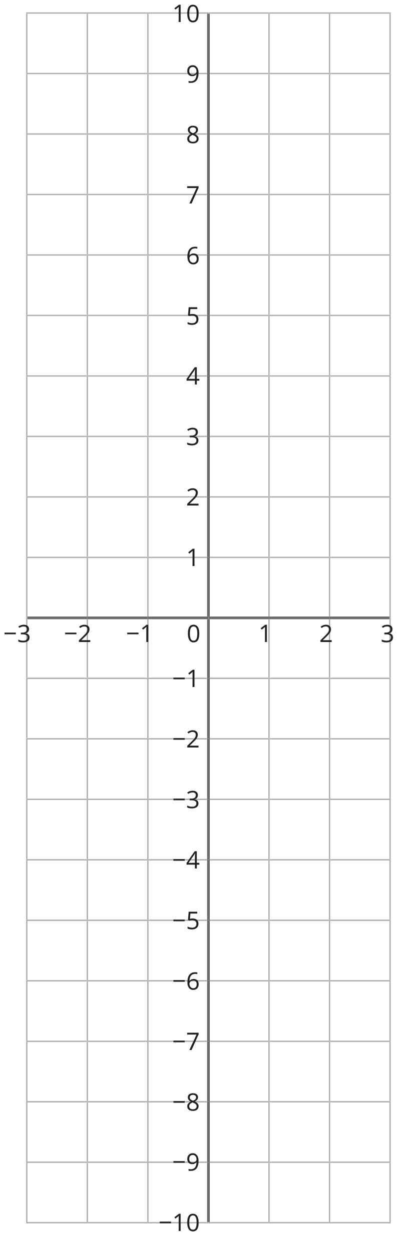
23. a. Complete the table of values for $y = -x^3$

x	−2	−1	0	1	2
y					

b. On the grid, draw the graph of $y = -x^3$ for $-2 \le x \le 2$



24. A straight line graph has the equation $y = 5x + 7$
Write down the coordinates of the y–intercept.

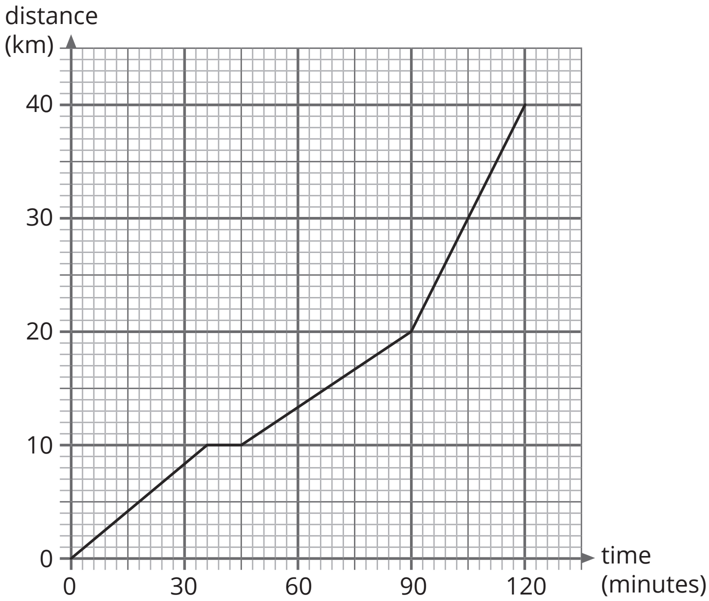
25. A straight line graph has the equation $y = 3x - 2$
Write down the value of the gradient.

26. A straight line graph has the equation $2y - 3x = 4$
Write down the value of the gradient.

27. A straight line graph has the equation $y = 2x + 1$
Does the point (2, 4) lie on this line? You must show your working

28. Write down an equation for a line parallel to the line with equation $y = 3x + 5$

29. The distance-time graph shows Ava's journey to visit her friend.



a. Ava left home at 9am. At what time did Ava stop for a rest?

b. Work out Ava’s average speed for the entire journey, giving your answer in kilometres per hour.

c. During which period was Ava travelling the fastest?

Foundation and Higher GCSE Exam-Style Questions

Algebra 4-5 Answers

1. Fully simplify:

   a. $r \times r \times r \times r$

   r^4

   Be careful: a common mistake here is to write 4r. This actually means r + r + r + r

   b. $3a + 4a^2 + 2a - a^2$

   $5a + 3a^2$

   We only add like terms. a^2 and a are from different "families" so we can't combine these terms.

   c. $(2p^5q)^3$

   $2^3 \times p^{5 \times 3} \times q^3 = 8p^{15}q^3$

   Make sure you apply the power to everything in the bracket.

   d. $\frac{3x^2y}{6xy^2}$

   $\frac{3}{6} = \frac{1}{2}$

   $x^2 \div x = x$

   $y \div y^2 = y^{-1}$ or $\frac{1}{y}$

   $\frac{x}{2y}$

2. a. $y = 8n + 11$

   Work out the value of y when $n = 5$

   $y = 8 \times 5 + 11 = 51$

   Remember: $8n$ means 8 times n.

   b. $y = 3n - 4$

   Work out the value of n when $y = 8$

   Substitute $y = 8$ into the formula and then solve for n: $8 = 3n - 4$

   Add 4 to both sides: $12 = 3n$

   Divide by 3: $4 = n$

   Or $n = 4$

3. $v = ut + \frac{1}{2}at^2$

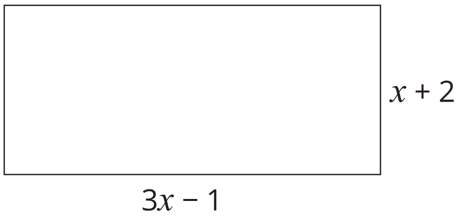
   Find the value of v when $u = 0$, $a = 5$ and $t = 3$

   $v = 0 \times 3 + \frac{1}{2} \times 5 \times 3^2 = 22.5$

   BIDMAS or BODMAS tell us to always square before multiplying.

BEYOND MATHS

4. The diagram shows a rectangle. All dimensions are in centimetres.



a. Write an expression in terms of x for the perimeter of the rectangle. Give your answer in its simplest form.

$x + 2 + x + 2 + 3x - 1 + 3x - 1 = (8x + 2)$cm

Don't forget to include the 'unlabelled' sides of the rectangle.

b. Given that the perimeter of the rectangle is 54cm, find the value of x.

$8x + 2 = 54$

Subtract 2 from both sides: $8x = 52$

Divide by 8: $x = 6.5$cm

Don't worry if you don't know 52 ÷ 8 by heart. You can always write it as a fraction and simplify as far as you can.

5. Alexis has n marbles. Ben has three more marbles than Alexis. Carly has twice the amount of marbles that Alexis and Ben have between them. Write an expression in terms of n for the total number of marbles that the three children have. Give your answer in its simplest form.

Alexis: n

Ben: $n + 3$

Carly: $2(n + n + 3) = 4n + 6$

Total: $n + n + 3 + 4n + 6 = 6n + 9$

6. Solve:

a. $3x = 15$

Divide by 3: $x = 5$

b. $x - 7 = 11$

Add 7: $x = 18$

c. $\frac{x}{4} = 8$

Multiply by 4: $x = 32$

d. $3(2x + 1) = 12$

Expand the brackets first: $6x + 3 = 12$

Subtract 3: $6x = 9$

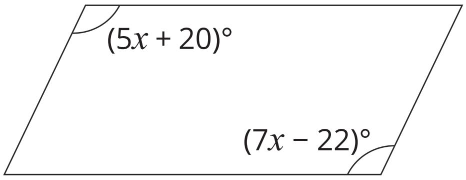
Divide by 6: $x = 1.5$ or $\frac{3}{2}$

You could have also divided by 3 first!

e. $5x - 1 = 2 - x$

When x is on both sides, always deal with the smallest value of x first.
Add x to both sides: $6x - 1 = 2$
Add 1: $6x = 3$
Divide by 6: $x = 0.5$

7. The diagram shows a parallelogram. Find the size of the smallest angle in the parallelogram.



Opposite angles in a parallelogram are equal: $5x + 20 = 7x - 22$
Subtract $5x$ from both sides: $20 = 2x - 22$
Add 22: $42 = 2x$
Divide by 2: $21 = x$
$x = 21$
Substitute this value into one of the expressions: $5 \times 21 + 20 = 125$
The smaller and larger angles are co-interior/supplementary/allied so they add to 180˚.
The smaller angle is 180 − 125 = 55˚

8. Rearrange to make x the subject:

a. $p = 3x + r$

Subtract r from both sides: $p - r = 3x$
Divide by 3: $x = \frac{p - r}{3}$
Remember: if you can solve equations, you can change the subject. Just perform inverse (opposite) operations.

b. $y = \frac{1}{2}x + 5$

Subtract 5 from both sides: $y - 5 = \frac{1}{2}x$
Multiply by 2: $x = 2(y - 5)$ or $x = 2y - 10$
Remember: dividing by $\frac{1}{2}$ is the same as multiplying by 2.
The answer is not $x = 2y - 5$

c. $A = \pi x^2$

Divide by π: $x^2 = \frac{A}{\pi}$
Square root both sides: $x = \sqrt{\frac{A}{\pi}}$
Make sure the square root symbol clearly covers the whole fraction.

9. Expand and simplify:

a. $4x(x + 7)$

Multiply everything on the inside of the bracket by $4x$.

$\mathbf{4x^2 + 28x}$

b. $3(2p + 1) - 2(p - 7)$

$\mathbf{6p + 3 - 2p + 14 = 4p + 17}$

Be careful: the + 14 comes from −2 × −7

c. $(y + 5)(y - 2)$

Use F.O.I.L. or the grid method and don't forget to simplify:

$\mathbf{y^2 + 5y - 2y - 10 = y^2 + 3y - 10}$

d. $(w + 2)^2$

Start by writing this out fully:

$\mathbf{(w + 2)(w + 2) = w^2 + 2w + 2w + 4}$

$\mathbf{= w^2 + 4w + 4}$

10. Factorise fully:

a. $8x - 12$

Find the highest common factor of $8x$ and −12. It's 4, so that goes on the outside of the bracket: $4(2x - 3)$

b. $9p^2 + 6p$

$\mathbf{3p(3p + 2)}$

A common mistake here, is to think that the answer is $3(3p^2 + 2p)$ but this isn't fully factorised as the terms inside the bracket still have a common factor of p.

c. $8ab + 10b + 12ab^2$

The highest common factor is $2b$.

$\mathbf{2b(4a + 5 + 6ab)}$

d. $x^2 + 9x + 20$

Find two numbers that multiply to make 20 and add to make 9.

$\mathbf{(x + 5)(x + 4)}$

e. $y^2 + 3y - 18$

Find two numbers that multiply to make −18 and add to make 3.

The factors of −18 are: 1 and −18, −1 and 18, −2 and 9, 2 and −9, −3 and 6, 3 and −6.

$\mathbf{(y + 6)(y - 3)}$

f. $4x^2 - 9$

This is called the "difference of two squares". The trick is to spot that both terms are squares and they are separated by a subtraction symbol.

$\mathbf{\sqrt{(4x^2)} = 2x, \sqrt{9} = 3}$

$\mathbf{(2x + 3)(2x - 3)}$

11. Solve the simultaneous equations:

a. $3x + y = 8$
$x + y = 2$

Here, we have the same coefficient of y in both equations.
Use the SSS rule – if the signs are the same, you subtract.
Here, the ys have the same sign, so we subtract everything in the equations.
$2x = 6$
Solve this equation by dividing by 2: $x = 3$
Finally, substitute this value back into either of the original equations. The second is easier: $3 + y = 2$
Solve by subtracting 3 from both sides: $y = -1$
$x = 3, y = -1$
Remember: you can check your answer by substituting into the other equation.

b. $4x + 3y = 8$
$2x - y = -1$

This time, we don't have the same number of xs or ys. So we multiply the second equation by 3: $6x - 3y = -3$
This time, the signs of the coefficient of y are different, so instead of subtracting we add the equations: $10x = 5$
Divide by 10: $x = 0.5$
Substitute into the first equation: $2 + 3y = 8$
Subtract 2: $3y = 6$
Divide by 3: $y = 2$
$x = 0.5, y = 2$

12. Two families are going to the cinema.
Ben buys two adult tickets and two child tickets and pays a total of £21.20.
Charlie buys one adult ticket and three child tickets and pays a total of £17.60.
Work out the cost of buying one adult ticket and one child ticket.

We need to form a pair of simultaneous equations and solve as normal.
Let a be the cost of an adult ticket in pence and c be the cost of a child ticket in pence. (You could choose pounds but, by calculating in pence, you don't need to worry about decimals.)
$2a + 2c = 2120$
$a + 3c = 1760$
Multiply the second equation by 2: $2a + 6c = 3520$
Subtract the first equation from the third: $4c = 1400$
Divide by 4: $c = 350$
Substitute into the second equation: $a + 1050 = 1760$
Subtract 1050: $a = 710$
adult ticket = £7.10, child ticket = £3.50
total = £10.60

13. A linear sequence has n^{th} term $4n - 3$
Work out the value of the eighth term in the sequence.

4 × 8 − 3 = 29

14. The first three terms of a geometric sequence are 400, 200, 100.
Write down the next two terms of this sequence.

The terms are halving, or being multiplied by $\frac{1}{2}$, each time.
50, 25

15. Find the n^{th} terms for the linear sequences whose first four terms are:

a. 3, 6, 9, 12

The common difference is 3 and the zero term (the term before the first term) is zero.
$3n$

b. 5, 11, 17, 23

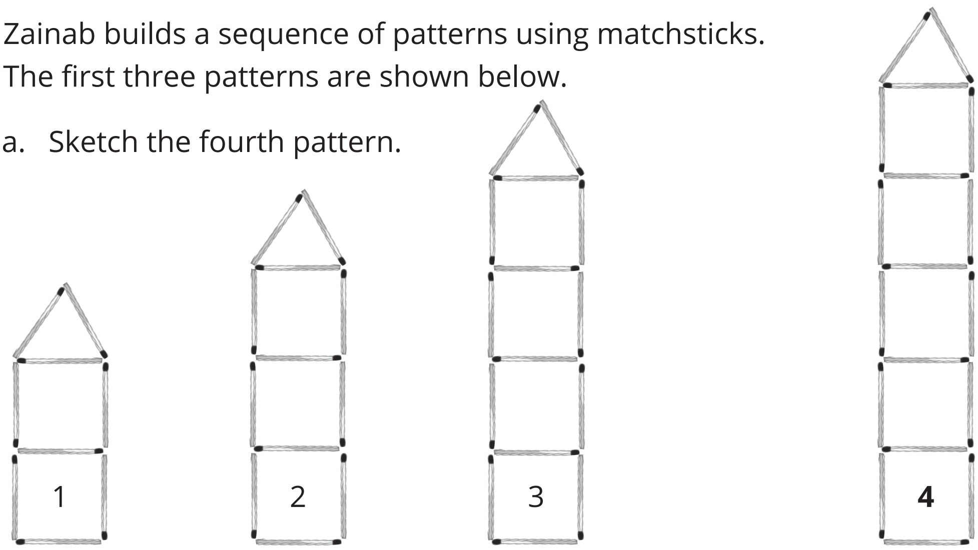
The common difference is 6 and the zero term (the term before the first term) is −1.
$6n - 1$

c. 12, 10, 8, 6

The common difference is −2 and the zero term (the term before the first term) is 14.
−2n + 14

16. Zainab builds a sequence of patterns using matchsticks.
The first three patterns are shown below.

a. Sketch the fourth pattern.



b. Work out how many matchsticks she would need for the 20^{th} pattern.

You could continue the pattern but it's sensible to find the n^{th} term.

The number of matchsticks is given by 9, 12, 15, 18 and so on. There is a common difference of 3 and a zero term of 6.

The n^{th} term is $3n + 6$

The 20^{th} term is 3 × 20 + 6 = 66

17. Work out the first three terms of the sequence whose n^{th} term is $10 - n^2$

The first term is when $n = 1$, the second is when $n = 2$ and the third is when $n = 3$.

$10 - 1^2 = 9$

$10 - 2^2 = 6$

$10 - 3^2 = 1$

9, 6, 1

18. The first four terms of a Fibonacci sequence are 1, 1, 2, 3. Work out the value of the 7^{th} term in this sequence.

13

The next term is found by adding the previous two.

The full sequence is 1, 1, 2, 3, 5, 8, 13, ...

19. a. Complete the table of values for $y = 3x - 1$

x	−2	−1	0	1	2
y	−7	**−4**	**−1**	2	**5**

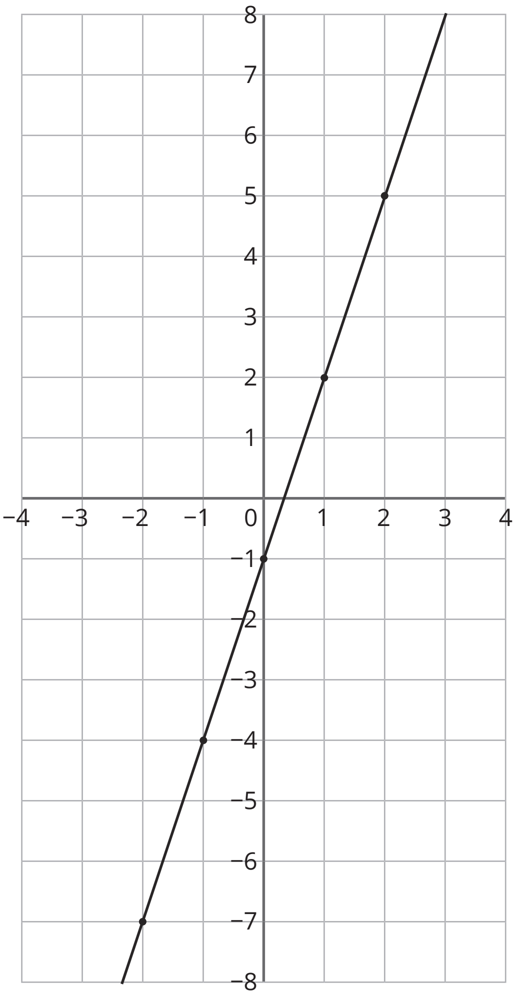
$3 \times -1 - 1 = -4$

$3 \times 0 - 1 = -1$

$3 \times 2 - 1 = 5$

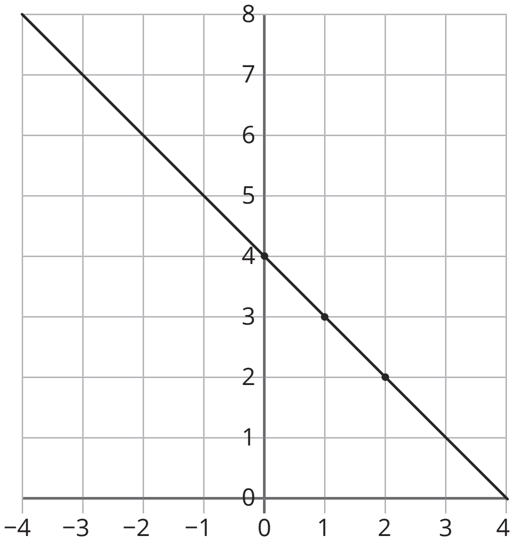
Look for a pattern. Here, the numbers are going up in 3s.

b. On the grid, draw the graph of $y = 3x - 1$ for $-2 \leq x \leq 2$



When the equation is of the form $y = mx + c$, always check that you have drawn a straight line.

20. On the grid, draw the graph of $x + y = 4$ for $-2 \leq x \leq 2$



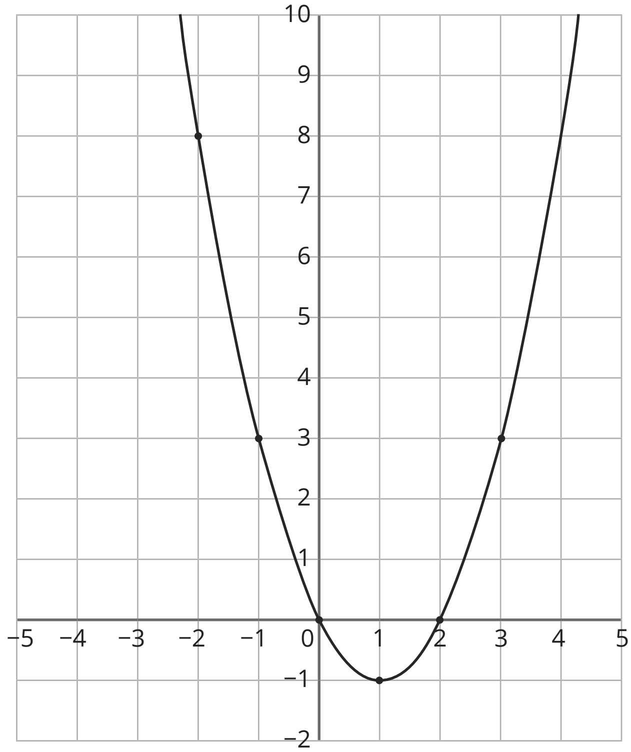
Don't be afraid to draw a table of your own. Some coordinates that you could choose are:
$x = 0, y = 4$,
$x = 1, y = 3$,
$x = 2, y = 2$.
Notice how each pair of coordinates adds to make 4.

21. a. Complete the table of values for $y = x^2 - 2x$

x	−2	−1	0	1	2	3
y	**8**	3	**0**	**−1**	0	**3**

$(-2)^2 - 2 \times -2 = 8$
$(0)^2 - 2 \times 0 = 0$
$(1)^2 - 2 \times 1 = -1$
$(3)^2 - 2 \times 3 = 3$

b. On the grid, draw the graph of $y = x^2 - 2x$ for $-2 \leq x \leq 3$



Quadratic graphs are usually symmetrical. Watch out for this as a way to check your work.

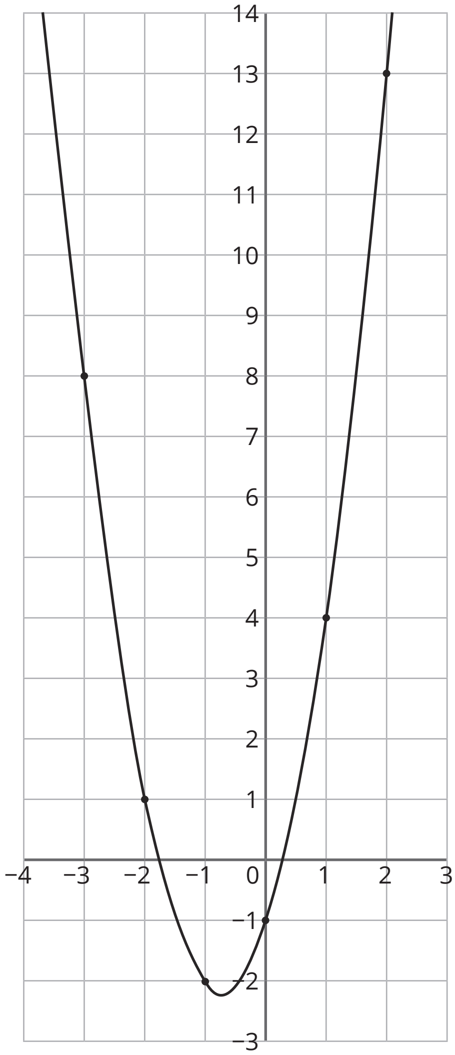
c. Write down the coordinates of the turning point of this graph.

This is the point at which the graph changes direction: (1, −1)

22. a. Complete the table of values for $y = 2x^2 + 3x - 1$

x	−3	−2	−1	0	1	2
y	8	**1**	**−2**	**−1**	4	**13**

b. On the grid, draw the graph of $y = 2x^2 + 3x - 1$ for $-3 \le x \le 2$



$2(-2)^2 + 3(-2) - 1 = 1$
$2(-1)^2 + 3(-1) - 1 = -2$
$2(0)^2 + 3(0) - 1 = -1$
$2(2)^2 + 3(2) - 1 = 13$

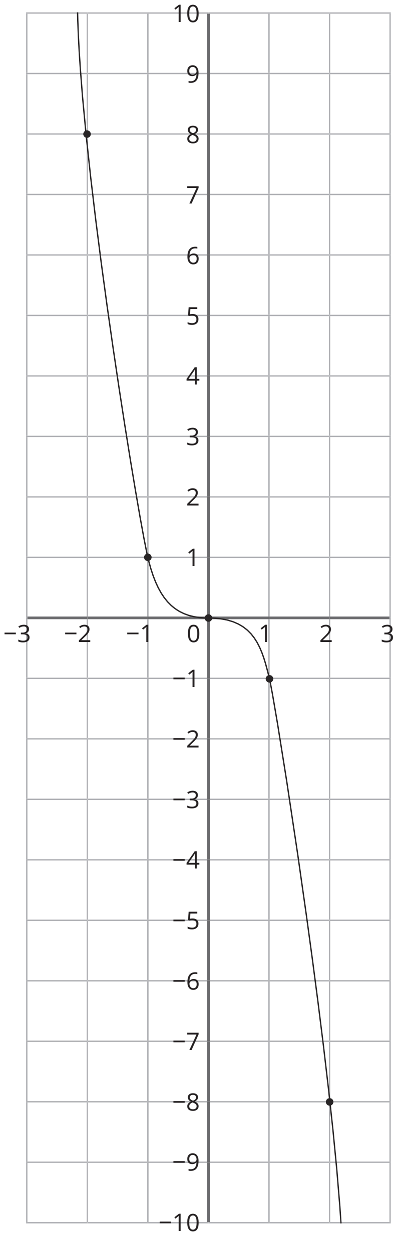
c. Use your graph to estimate all the solutions to the equation $2x^2 + 3x - 1 = 0$

This is the point at which the graph crosses the x-axis.
Approximately $x = -1.8$ and $x = 0.3$

23. a. Complete the table of values for $y = -x^3$

x	−2	−1	0	1	2
y	**8**	**1**	**0**	**−1**	**−8**

b. On the grid, draw the graph of $y = -x^3$ for $-2 \le x \le 2$



$\mathbf{-(-2)^3 = 8}$

$\mathbf{-(-1)^3 = 1}$

$\mathbf{-(0)^3 = 0}$

$\mathbf{-(1)^3 = -1}$

$\mathbf{-(2)^3 = -8}$

24. A straight line graph has the equation $y = 5x + 7$
Write down the coordinates of the y–intercept.

The general form is $y = mx + c$, where m is the gradient and c is the y-intercept. The y-intercept is therefore 7, but we need to write this in coordinate form.

(0, 7)

25. A straight line graph has the equation $y = 3x - 2$
Write down the value of the gradient.

The general form is $y = mx + c$, where m is the gradient and c is the y-intercept.
The gradient is therefore 3.

26. A straight line graph has the equation $2y - 3x = 4$
Write down the value of the gradient.

The general form is $y = mx + c$, where m is the gradient and c is the y-intercept. We need to rearrange this equation, first, by adding $3x$ to both sides; then, dividing through by 2.

$y = 1.5x + 2$
The gradient is 1.5 or $\frac{3}{2}$.

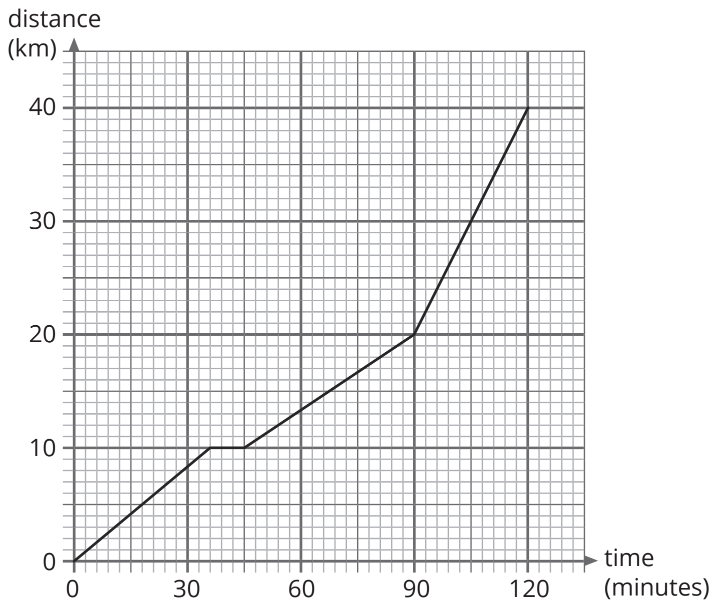
27. A straight line graph has the equation $y = 2x + 1$
Does the point (2, 4) lie on this line? You must show your working

Substitute $x = 2$ into the equation $y = 2x + 1$
$y = 2 \times 2 + 1 = 5$
No, (2, 4) does not lie on the line. When $x = 2$, $y = 5$

28. Write down an equation for a line parallel to the line with equation $y = 3x + 5$

When two lines are parallel, their gradients are equal.
$y = 3x + c$, where $c \neq 5$
E.g. $y = 3x + 1$

29. The distance-time graph shows Ava's journey to visit her friend.



a. Ava left home at 9am. At what time did Ava stop for a rest?

The scale on the x-axis is tricky. 10 squares represent 30 minutes so 1 square represents 3 minutes.

9:36am or 09:36

b. Work out Ava's average speed for the entire journey, giving your answer in kilometres per hour.

Speed = $\frac{\text{distance}}{\text{time}}$

40 ÷ 2 = 20km/h.

c. During which period was Ava travelling the fastest?

The speed is given by the gradient of the graph. The steeper the graph, the faster Ava is travelling.

Between 10:30am and 11am, or between 1 hour 30 minutes after setting off and 2 hours after setting off.